# A Production Lot Size Model for a product subject to deterioration 

J. Jagadeeswari, P. K. Chenniappan


#### Abstract

In this paper, we present a method for finding the optimal replenishment schedule for the production lot size model with deteriorating items where the deterioration is continuous in accordance with a general probability distribution under Last In First Out policy (LIFO) issuing policy.


Key Words - Deteriorating items, Economic Production Quantity, Last In First Out issuing policy, Perturbation technique, Exponential distribution, Weibull distribution.

## 1. Introduction

The classical dynamic lot sizing model of Wagner and Whitin (1958) and its extensions deal with the problem of finding the optimal replenishment policy for an item under the assumption that inventory can be carried for an indefinite number of periods. This assumption cannot be justified if one considers potentially obsolete or perishable products like camera films, blood, agricultural products, electronic gadgets, etc. These products cannot be used after a certain number of periods for one of the following reasons:

- The utility of the products drop to almost zero after a fixed time period (fixed life time due to physical or legal causes).
- The utility of the product decreases throughout the life time (e.g., exponential decay).
- The utility of the product drops to zero due to some external factor such as the failure of a special storage environment, a change in engineering design, etc.

Friedman and Hoch (1978) considered a model similar to Wagner and Whitin (1958). In that inventory levels are reviewed periodically and demand is assumed known. In addition, they assumed that a known fraction ( $0 \leq \mathrm{r}_{i} \leq 1$ ) of the units on hand of age $i$ survive into the next period. They also stated that 'the property that one only orders in periods in which starting stock is zero' no longer holds when perishability is allowed.

An EOQ model for items with a variable rate of deterioration, an infinite rate of production and no shortage was introduced by Covert and Philip (1973).

Extensive research has been done on fixed life time perishable products. An excellent review is provided by Nahmias (1982).

In this paper, an Economic Production Quantity model with Last In First Out (LIFO) issuing policy for
demand with items that deteriorate continuously in accordance with a general probability distribution for the lifetime of an item is developed.

## 2. Model Assumptions and notations

The inventory model presented in this paper is based on the following assumptions:

- A single item is held in stock.
- Demand rate $\lambda$ is known and constant.
- Production rate $P$ is finite and constant.
- Units are available for satisfying demand after their production.
- There is no repair or replacement of items that deteriorate during a cycle.
- The deterioration occurs only where the item is effectively in stock.
- The production rate $P$ is greater than the demand rate.
- Shortages are not allowed.
- The number of units is treated as a continuous variable.
- The time for an item to deteriorate follows probability density function (p.d.f) $f(t) \quad(t \geq 0)$ and cumulative distribution function $\mathrm{F}(\mathrm{t})=1-$ $R(t)$; so that, the instantaneous deterioration rate of an item is $\mathrm{D}(\mathrm{t})=\mathrm{f}(\mathrm{t}) /(1-\mathrm{F}(\mathrm{t}))=\mathrm{f}(\mathrm{t}) / \mathrm{R}(\mathrm{t}), \mathrm{t} \geq 0$.
- Last in first out (LIFO) principle is applied in satisfying demand.
- $\mathrm{I}_{\mathrm{t}}$ : Inventory level at time t
- T : Inventory cycle time.
- $\mathrm{T}_{1}$ : Time at which the inventory level is at maximum.


## 3. Mathematical Development

Figure (3.1) shows an inventory cycle for a finite production rate where T is a cycle time.


FIGURE 3.1

During the time interval $\left(0, \mathrm{~T}_{1}\right)$ production occurs at a constant rate of P units per unit time and demand occurs at a constant rate of $\lambda$ units per unit time. Due to LIFO policy, $(P-\lambda) \Delta t$ enters the inventory system during the time interval $(\mathrm{t}, \mathrm{t}+\Delta \mathrm{t})$ where $\mathrm{t} \leq \mathrm{T}_{1}$.

At time $t_{1}$ where $t \leq t_{1} \leq T_{1}$, the quantity $\quad(P-\lambda) \Delta t$ which entered the inventory during
$(t, t+\Delta t)$ reduces to $(P-\lambda) R\left(t_{1}-t\right) \Delta t$ due to deterioration.

This gives the inventory level at time $t_{1}, I_{t_{1}}$ as follows

$$
\begin{equation*}
I_{t_{1}}=\int_{0}^{t_{1}}(P-\lambda) \mathrm{R}\left(\mathrm{t}_{1}-\mathrm{t}\right) \mathrm{dt} \tag{3.1}
\end{equation*}
$$

During the time interval $\left(\mathrm{T}_{1}, \mathrm{~T}\right)$, there is no production and demand occurs at a constant rate of $\lambda$ units per unit time which is satisfied from the inventory accumulated during $\left(0, \mathrm{~T}_{1}\right)$.Thus, for the interval $\left(\mathrm{t}_{2}, \mathrm{t}_{2}+\Delta \mathrm{t}\right)$ where $\mathrm{T}_{1} \leq \mathrm{t}_{2} \leq \mathrm{T}, \lambda \Delta \mathrm{t}_{2}$ will be the demand.


FIGURE 3.2

Assume that the demand $\lambda \Delta t_{2}$ is satisfied with the items produced during $\left(t\left(t_{2}\right)-\Delta t, t\left(t_{2}\right)\right)$ shown in figure (3.2).

Thus, at time $\mathrm{t}_{2}$ the item produced during $\left(\mathrm{t}\left(\mathrm{t}_{2}\right), \mathrm{T}_{1}\right)$ is not in the inventory system because they already satisfied the demand occurred during $\left(t\left(t_{2}\right), t_{2}\right)$ due to LIFO principle . This gives

$$
\begin{equation*}
\lambda \Delta \mathrm{t}_{2}=(\mathrm{P}-\lambda) \mathrm{R}\left(\mathrm{t}_{2}-\mathrm{t}\right)(-\Delta \mathrm{t}) \tag{3.2}
\end{equation*}
$$

Therefore,

$$
\begin{equation*}
\lambda \frac{\mathrm{dt}_{2}}{\mathrm{dt}}=-(\mathrm{P}-\lambda) \mathrm{R}\left(\mathrm{t}_{2}-\mathrm{t}\right) \tag{3.3}
\end{equation*}
$$

If $R(t)$ is known, $t\left(t_{2}\right)$ can be found from Equation (3.3) with initial condition, $t=T_{1}$ at $t_{2}=T_{1}$ due to LIFO policy and the inventory level at $t_{2}$ will be given by

$$
\begin{equation*}
I_{t_{2}}=\int_{0}^{t\left(t_{2}\right)}(P-\lambda) \mathrm{R}\left(\mathrm{t}_{2}-\mathrm{y}\right) \mathrm{dy} \tag{3.4}
\end{equation*}
$$

### 3.1 CASES

From the general inventory level developed above, two particular cases are considered by taking the exponential distribution and the general Weibull distribution for the time to deterioration of an item.

### 3.1.1 Exponential distribution for the time to deterioration of an item

Let the p.d.f of the time to deterioration of an item be

$$
f(t)=\left\{\begin{array}{cc}
\alpha e^{(-\alpha t)} & , t \geq 0, \alpha>0 \\
0 & , \text { otherwise }
\end{array}\right\}
$$

For this distribution

$$
\begin{aligned}
F(t) & =\int_{0}^{t} f(t) d t \\
& =\int_{0}^{t} \alpha e^{(-\alpha t)} d t \\
& =1-e^{-\alpha t}
\end{aligned}
$$

Thus, $\mathrm{R}(\mathrm{t})=1-\mathrm{F}(\mathrm{t})=\mathrm{e}^{(-\alpha \mathrm{t})}$ and $D(t)=\frac{f(t)}{R(t)}=\alpha$.
With substitution of $e^{\left(-\alpha\left(t_{1}-t\right)\right)}$ into $\mathrm{R}\left(\mathrm{t}_{1}-\mathrm{t}\right)$ of Equation (3.1) and integrating we get,

$$
\begin{align*}
I_{t_{1}} & =\int_{0}^{t_{1}}(P-\lambda) e^{\left(-\alpha\left(t_{1}-t\right)\right)} d t \\
& =\frac{(P-\lambda)}{\alpha}\left[1-e^{\left(-\alpha t_{1}\right)}\right], 0 \leq t_{1} \leq T_{1} \tag{3.5}
\end{align*}
$$

Similarly from Equation (3.4) we get

$$
\begin{align*}
I_{t_{2}} & =\int_{0}^{t\left(t_{2}\right)}(P-\lambda) e^{\left(-\alpha\left(t_{2}-y\right)\right)} d y \\
& =\frac{(P-\lambda)}{\alpha} e^{\left(-\alpha t_{2}\right)}\left[e^{\alpha t}-1\right] \tag{3.6}
\end{align*}
$$

From Equation (3.3) we get

$$
\lambda \frac{d t_{2}}{d t}=-(P-\lambda) e^{\left(-\alpha\left(t_{2}-t\right)\right)}
$$

Solving the above equation we get

$$
\begin{equation*}
\frac{(P-\lambda) e^{\alpha t}}{\alpha}=\frac{-\lambda e^{\alpha t_{2}}}{\alpha}+C \tag{3.14}
\end{equation*}
$$

where $C$ is the constant of integration. Applying the boundary condition, $\mathrm{t}=\mathrm{T}_{1}$ at $\mathrm{t}_{2}=\mathrm{T}_{1}$, we get,

$$
\begin{equation*}
(P-\lambda) e^{\alpha t}=P e^{\alpha t_{1}}-\lambda e^{\alpha t_{2}} \tag{3.8}
\end{equation*}
$$

Substituting Equation (3.8) in equation (3.6) yields

$$
\begin{align*}
I_{t_{2}} & =\frac{1}{\alpha}\left\{e^{\left(-\alpha t_{2}\right)}\left(P e^{\alpha T_{1}}-\lambda e^{\alpha t_{2}}\right)-(P-\lambda) e^{\left(-\alpha t_{2}\right)}\right\} \\
& =\frac{1}{\alpha}\left\{P e^{\alpha\left(T_{1}-t_{2}\right)}-\lambda-(P-\lambda) e^{\left(-\alpha t_{2}\right)}\right\}-\cdots------- \tag{3.9}
\end{align*}
$$

### 3.1.2 General Weibull distribution for the time to deterioration of an item

Let the p.d.f of the time to deterioration of an item be

$$
f(t)=\left\{\begin{array}{cc}
\alpha \beta t^{\beta-1} e^{\left(-\alpha t^{\beta}\right)} & , t \geq 0, \alpha>0, \beta>0 \\
0 & , \text { otherwise }
\end{array}\right\}
$$

where $\alpha, \beta$ are some constants determined by the deterioration process. For this distribution,

$$
\begin{align*}
F(t) & =\int_{0}^{t} f(t) d t  \tag{3.16}\\
& =\int_{0}^{t} \alpha \beta t^{\beta-1} e^{\left(-\alpha t^{\beta}\right)} d t \\
& =1-e^{\left(-\alpha t^{\beta}\right)}
\end{align*}
$$

Thus, $R(t)=1-F(t)=e^{\left(-\alpha t^{\beta}\right)}$ and

$$
D(t)=\frac{f(t)}{R(t)}=\alpha \beta t^{\beta-1}
$$

$$
\alpha^{\frac{1}{\beta}} \beta x^{\left(\frac{\beta-1}{\beta}\right)}\left(\frac{\lambda-P}{\lambda} e^{(-x)}-1\right)=\frac{d x}{d t}
$$

Solving Equation (3.14) we get

$$
t\left(t_{2}\right)=-\int_{0}^{x}\left(\alpha^{\frac{1}{\beta}} \beta y^{\frac{\beta-1}{\beta}}\left(1+\frac{P-\lambda}{\lambda} e^{(-y)}\right)\right)^{-1} d y+C
$$

where C is the constant of integration. Applying boundary condition, $\mathrm{t}=\mathrm{T}_{1}$ at $\mathrm{t}_{2}=\mathrm{T}_{1}$, which in turn implies $\mathrm{x}=0$ at $\mathrm{t}=$ T, we get

$$
\begin{equation*}
t\left(t_{2}\right)=-\int_{0}^{x}\left(\alpha^{\frac{1}{\beta}} \beta y^{\frac{\beta-1}{\beta}}\left(1+\frac{P-\lambda}{\lambda} e^{(-y)}\right)\right)^{-1} d y+T_{1} \tag{3.15}
\end{equation*}
$$

Equation (3.15) is a transcendental equation and solving with respect to $t\left(t_{2}\right)$ is very difficult. One way to obtain an approximate solution of $t\left(t_{2}\right)$ is to solve equation (3.12) under assumption that $\alpha \leq 1$.

## Approximation

Let $\mathrm{u}=t_{2}-t$ and $\mathrm{v}=\mathrm{P}-\lambda$. Then Equation (3.12) becomes

$$
\lambda\left(\frac{d u}{d t}+1\right)=-v e^{\left(-\alpha u^{\beta}\right)}
$$

Solving equation (3.16) we get

$$
\begin{aligned}
& \frac{(-\lambda)}{\lambda+v e^{(\lambda u \beta)}} d u=d t \\
& \int_{0}^{u} \frac{(-\lambda) d u^{\prime}}{\lambda+v e^{\left(-\lambda u^{\prime \beta}\right)}}=t+C
\end{aligned}
$$

where C is the constant of integration. Applying the boundary condition, at $\mathrm{t}_{2}=\mathrm{T}_{1}, \mathrm{t}=\mathrm{T}_{1}$, which in turn implies, at $u=0, t=T_{1}$, we get

$$
\begin{equation*}
\int_{0}^{u} \frac{(-\lambda) d u^{\prime}}{\lambda+v e^{\left(-\lambda u^{\prime \beta}\right)}}=t-T_{1^{-}} \tag{3.17}
\end{equation*}
$$

To solve Equation (3.17) first consider the L.H.S. Expanding the denominator using the series form of the exponential and ignoring terms with third and higher order powers of $\alpha$, we get

$$
\begin{aligned}
\lambda+v e^{\left(-\lambda u^{\prime \beta}\right)} & =\lambda+v-v \alpha u^{\prime \beta}+\frac{v \alpha^{2} u^{\prime 2 \beta}}{2}+o\left(\alpha^{3}\right) \\
& =(\lambda+v)\left(1-\frac{v \alpha u^{\prime \beta}-\frac{v \alpha^{2} u^{\prime 2 \beta}}{2}}{\lambda+v}+o\left(\alpha^{3}\right)\right)
\end{aligned}
$$

$$
\alpha \beta\left(\frac{x}{\alpha}\right)^{\frac{\beta-1}{\beta}}\left(\frac{-(P-\lambda)}{\lambda} e^{(-x)}-1\right)=\frac{d x}{d t}
$$

Hence the integrand in Equation (3.17) becomes

$$
\begin{aligned}
\frac{(-\lambda)}{\lambda+v e e^{\left(-\alpha u^{\prime \beta}\right)}} & =\frac{-\lambda}{\lambda+v}\left(1-\frac{v \alpha u^{, \beta}-\frac{v \alpha^{2} u^{\prime 2 \beta}}{2}}{\lambda+v}+o\left(\alpha^{3}\right)\right)^{-1} \\
& =\frac{-\lambda}{\lambda+v}\left(1+\frac{v}{\lambda+v} \alpha u^{\prime \beta}+\frac{v(v-\lambda)}{2(\lambda+v)^{2}} \alpha^{2} u^{, 2 \beta}+o\left(\alpha^{3}\right)\right)
\end{aligned}
$$

Thus, the L.H.S. of Equation (3.17) becomes

$$
\left.\begin{array}{rl}
\int_{0}^{u} \frac{(-\lambda) d u^{\prime}}{\lambda+v e^{\left(-\alpha u^{\prime \beta}\right)}}= & \frac{-\lambda}{\lambda+v} \int_{0}^{u}\left(1+\frac{v}{\lambda+v} \alpha u^{, \beta}+\frac{v(v-\lambda)}{2(\lambda+v)^{2}} \alpha^{2} u^{\prime 2 \beta}\right. \\
& \left.+o\left(\alpha^{3}\right)\right) d u^{\prime}
\end{array}\right] \begin{aligned}
= & \frac{-\lambda}{\lambda+v}\left(1+\frac{v \alpha u^{\prime \beta+1}}{(\lambda+v)(\beta+1)}+\frac{v(v-\lambda) \alpha^{2} u^{, 2 \beta+1}}{2(\lambda+v)^{2}(2 \beta+1)}+o\left(\alpha^{3}\right)\right)_{0}^{u} \\
= & \frac{-\lambda u}{\lambda+v}\left(1+\frac{v \alpha u^{\beta}}{(\lambda+v)(\beta+1)}+\frac{v(v-\lambda) \alpha^{2} u^{2 \beta}}{2(\lambda+v)^{2}(2 \beta+1)}+o\left(\alpha^{3}\right)\right)
\end{aligned}
$$

Thus Equation (3.17) becomes

$$
\begin{equation*}
\frac{-\lambda u}{\lambda+v}\left(1+\frac{v \alpha u^{\beta}}{(\lambda+v)(\beta+1)}+\frac{v(v-\lambda) \alpha^{2} u^{2 \beta}}{2(\lambda+v)^{2}(2 \beta+1)}+o\left(\alpha^{3}\right)\right)=t-T_{1}-\cdots \tag{3.18}
\end{equation*}
$$

Let $t=g\left(t_{2}\right)+\alpha g_{1}\left(t_{2}\right)+\alpha^{2} g_{2}\left(t_{2}\right)+o\left(\alpha^{3}\right)$ then u becomes

$$
\begin{equation*}
u=t_{2}-g_{0}-\alpha g_{1}-\alpha^{2} g_{2}+o\left(\alpha^{3}\right) \tag{3.19}
\end{equation*}
$$

Substituting Equation (3.19) in (3.18) the L.H.S. of Equation (3.18) becomes

$$
\begin{gathered}
\text { LHS }=\frac{-\lambda}{\lambda+v}\left(t_{2}-g_{0}-\alpha g_{1}-\alpha^{2} g_{2}+o\left(\alpha^{3}\right)\right)\left\{( \frac { v \alpha } { ( \lambda + v ) ( \beta + 1 ) } ) \left(t_{2}-\right.\right. \\
\left.g_{0}-\alpha g_{1}-\alpha^{2} g_{2}\right)^{\beta}+\frac{v(v-\lambda) \alpha^{2}}{2(\lambda+v)^{2}(2 \beta+1)}\left(t_{2}-g_{0}-\right. \\
\left.\left.\alpha g_{1}-\alpha^{2} g_{2}\right)^{2 \beta}+o\left(\alpha^{3}\right)\right\} \\
=\frac{-\lambda}{\lambda+v}\left(t_{2}-g_{0}-\alpha g_{1}-\alpha^{2} g_{2}+o\left(\alpha^{3}\right)\right)\left\{( \frac { v \alpha } { ( \lambda + v ) ( \beta + 1 ) } ) \left(t_{2}-\right.\right. \\
\left.\left.g_{0}\right)^{\beta}\left(1-\frac{\alpha g_{1}+\alpha^{2} g_{2}}{t_{2}-g_{0}}\right)+\frac{v(v-\lambda) \alpha^{2}}{2(\lambda+v)^{2}(2 \beta+1)}\left(t_{2}-g_{0}\right)^{2 \beta}+o\left(\alpha^{3}\right)\right\}
\end{gathered}
$$

Now using the approximation formula $(1-x)^{\beta}=1-\beta x$, we get

LHS $=\frac{-\lambda}{\lambda+v}\left(t_{2}-g_{0}-\alpha g_{1}-\alpha^{2} g_{2}+o\left(\alpha^{3}\right)\right)\left\{\left(\frac{v \alpha}{(\lambda+v)(\beta+1)}\right)\left(t_{2}-\right.\right.$ $\left.\left.g_{0}\right)^{\beta}\left(1-\frac{\alpha g_{1}+\alpha^{2} g_{2}}{t_{2}-g_{0}}\right)+\frac{v(v-\lambda) \alpha^{2}}{2(\lambda+v)^{2}(2 \beta+1)}\left(t_{2}-g_{0}\right)^{2 \beta}+o\left(\alpha^{3}\right)\right\}$

With this equation (3.18) becomes

$$
\begin{gather*}
\frac{-\lambda}{\lambda+v}\left(t_{2}-g_{0}-\alpha g_{1}-\alpha^{2} g_{2}+o\left(\alpha^{3}\right)\right)\left\{( \frac { v \alpha } { ( \lambda + v ) ( \beta + 1 ) } ) \left(t_{2}-\right.\right. \\
\left.\left.g_{0}\right)^{\beta}\left(1-\frac{\alpha \beta g_{1}}{t_{2}-g_{0}}\right)+\frac{v(v-\lambda) \alpha^{2}}{2(\lambda+v)^{2}(2 \beta+1)}\left(t_{2}-g_{0}\right)^{2 \beta}+o\left(\alpha^{3}\right)\right\} \\
=g_{0}+\alpha g_{1}+\alpha^{2} g_{2}-T_{1}+o\left(\alpha^{3}\right) \tag{3.20}
\end{gather*}
$$

First equating the constant terms we get

$$
\begin{gather*}
g_{0}-T_{1}=\frac{-\lambda}{\lambda+v}\left(t_{2}-g_{0}\right) \\
=\frac{-\lambda}{P}\left(t_{2}-g_{0}\right) \\
g_{0}=\frac{1}{P-\lambda}\left(P T_{1}-\lambda t_{2}\right) \tag{3.21}
\end{gather*}
$$

Now, equating the terms with $\alpha$ we egt

$$
\begin{align*}
g_{1} & =\frac{-\lambda v\left(t_{2}-g_{0}\right)^{\beta+1}}{(\lambda+v)^{2}(\beta+1)}+\frac{\lambda}{\lambda+v} g_{1} \\
& =\frac{-\lambda(P-\lambda)\left(t_{2}-g_{0}\right)^{\beta+1}}{P^{2}(\beta+1)}+\frac{\lambda}{P} g_{1} \\
g_{1} & =\frac{-\lambda}{P(\beta+1)}\left(t_{2}-g_{0}\right)^{\beta+1} \tag{3.22}
\end{align*}
$$

Finally, equating the terms with $\alpha^{2}$ we get

$$
\begin{align*}
& g_{2}=\frac{\lambda v \beta g_{1}}{(\lambda+v)^{2}(\beta+1)}\left(t_{2}-g_{0}\right)^{\beta} \\
& -\frac{\lambda \mathrm{v}(\mathrm{v}-\lambda)}{2(\lambda+v)^{3}(2 \beta+1)}\left(t_{2}-g_{0}\right)^{2 \beta+1} \\
& +\frac{\lambda \operatorname{vg}_{1}}{(\lambda+v)^{2}(\beta+1)}\left(t_{2}-g_{0}\right)^{\beta}+\frac{\lambda}{\lambda+v} g_{2} \\
& =\frac{\lambda(P-\lambda) \beta \mathrm{g}_{1}}{P^{2}(\beta+1)}\left(t_{2}-g_{0}\right)^{\beta} \\
& +\frac{\lambda(\mathrm{P}-\lambda)(\mathrm{P}-2 \lambda)}{2 \mathrm{P}^{3}(2 \beta+1)}\left(t_{2}-g_{0}\right)^{2 \beta+1} \\
& +\frac{\lambda(P-\lambda) \mathrm{g}_{1}}{P^{2}(\beta+1)}\left(t_{2}-g_{0}\right)^{\beta}+\frac{\lambda}{P} g_{2} \\
& g_{2}=\frac{\left(t_{2}-g_{0}\right)^{\beta}}{P}\left(\frac{\lambda \beta g_{1}}{\beta+1}+\frac{\lambda g_{1}}{\beta+1}-\frac{\lambda(P-2 \lambda)\left(t_{2}-g_{0}\right)^{\beta+1}}{2 P(2 \beta+1)}\right) \\
& g_{2}=\frac{\left(t_{2}-g_{0}\right)^{\beta}}{P}\left(\lambda g_{1}-\frac{\lambda(P-2 \lambda)\left(t_{2}-g_{0}\right)^{\beta+1}}{2 P(2 \beta+1)}\right) \tag{3.23}
\end{align*}
$$

With the perturbation technique, it is theoretically possible to obtain an approximate value of $t$ to any desired accuracy using higher powers of $\alpha$. Table(3.1) is the tabulated results of $t\left(t_{2}\right)$ from example problems to compare the approximate formula of $\mathrm{t}\left(\mathrm{t}_{2}\right), \mathrm{t}\left(\mathrm{t}_{2}\right)=\mathrm{g}_{0}\left(\mathrm{t}_{2}\right)+\alpha \mathrm{g}_{1}\left(\mathrm{t}_{2}\right)+$ $\alpha^{2} g_{2}\left(\mathrm{t}_{2}\right)$ where $\mathrm{g}_{1}\left(\mathrm{t}_{2}\right)$ is given by Equations (3.21), (3.22) and (3.23) respectively with the exact values calculated from Equation (3.8) when $\beta=1$.

Table 3.1: Calculated values of $\mathrm{t}\left(\mathrm{t}_{2}\right)$ with $\mathrm{P}=12, \lambda=8$ and $\mathrm{T}_{1}=5$.
E : Exact value from equation (3.8)
A: Value from $t=g_{0}+\alpha g_{1}\left(\mathrm{t}_{2}\right)+\alpha^{2} \mathrm{~g}_{2}\left(\mathrm{t}_{2}\right)$

Equating terms with the same power of $\alpha$ :

| Cases | E | $\alpha=0.1$, <br> $\beta=0.1$ | $\alpha=0.1$, <br> $\beta=0.5$ | $\alpha=0.1$, <br> $\beta=1.5$ |
| :--- | :--- | :--- | :--- | :--- |
|  |  | A | A | A |
| 5.0 | 5.0000 | 5.0000 | 5.0000 | 5.0000 |
| 5.5 | 3.9181 | 3.9188 | 3.9129 | 3.9189 |
| 6.0 | 2.6385 | 2.65 | 2.7474 | 2.4628 |
| 6.5 | 1.0893 | 1.1563 | 1.1527 | 0.234 |
| CYCLE TIME | 6.8637 | 6.8438 | 7.0938 | 6.5625 |

Table 3.1

We notice that the results from approximate formula are in good agreement with those by exact values. Once $t\left(t_{2}\right)$ is found, the inventory level can be calculated with Equation (3.10) and Equation (3.11).

To illustrate the use of the formula an approximate optimum production quantity is found in the production system where no shortage is permitted. Then total cost (TC) during a cycle time, T , consists of setup cost, production cost and holiday cost. Thus

$$
T C=C_{3}+C P T_{1}+C_{1}\left(\int_{0}^{T_{1}} I_{t_{1}} d t_{1}+\int_{T_{1}}^{T} I_{t_{2}} d t_{2}\right)
$$

$\mathrm{T}_{1}{ }^{*}$ which minimizes the total cost cannot be derived in a closed form. But an approximate optimal solution can be found by a numerical calculation using the formula developed.

## 4. Conclusion

In this problem, inventory level in a production quantity model for items that deteriorate continuously in accordance with a general probability distribution has been developed. When the rate of deterioration is variable, the items which have entered inventory at different times have a different rate of deterioration, since the amount deteriorated during a given interval depends on how long an item has been in stock. To overcome this difficulty we assume the Last In First Out(LIFO) issuing policy. Weibull and exponential distribution for the time to deterioration of an item are considered.

Due to difficulty in solving $t\left(t_{2}\right)$ in Eq. (3.15) an approximation formula using perturbation techniques is developed.

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## About Authors

## J. Jagadeeswari

Sri Ramakrishna Engineering College
Coimbatore - 641030, Tamilnadu, India
jidhiva13@gmail.com

## P. K. Chenniappan

Department of Mathematics
Government Arts College (Autonomous)
Coimbatore - 641018, Tamilnadu, India
pkchenniappan@gmail.com


